

## ON THE UNIFORM CONVERGENCE AND INTEGRABILITY OF SPECIAL TRIGONOMETRIC INTEGRALS

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*Abstract.* Necessary and sufficient conditions for the uniform convergence of trigonometric Fourier integrals are well-established when admissible monotone or general monotone functions are considered. In this paper, we generalize these main results by giving such conditions for the uniform convergence of sine and cosine integrals  $\int_0^\infty f_1(x) \sin(ux^p) dx$  and  $\int_0^\infty f_2(x) \cos(ux^p) dx$  in case of admissible general monotone functions  $f_1$  and  $f_2$ . Moreover, we give necessary and sufficient conditions for the  $L^q$ -integrability with the power weights of these integrals when non-negative functions  $f_1$  and  $f_2$  belong to the class  $\overline{GM}_{p\theta}$ .

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### REFERENCES

- [1] J. BENEDETTO AND H. HEINIG, *Weighted Fourier Inequalities: New Proofs and Generalizations*, J. Fourier Anal. Appl., **9** (2003), 1–37.
- [2] A. DEBERNARDI, *Uniform convergence of sine transforms of general monotone functions*, Math. Nachr., **290** (2017), 2815–2825.
- [3] M. DYACHENKO, E. LIFLYAND AND S. TIKHONOV, *Uniform convergence and integrability of Fourier integrals*, J. Math. Anal. Appl., **372** (2010), 328–338.
- [4] M. DYACHENKO AND S. TIKHONOV, *Convergence of trigonometric series with general monotone coefficients*, C. R. Acad. Sci. Paris, Ser. I, **345** (2007), 123–126.
- [5] M. DYACHENKO AND S. TIKHONOV, *Integrability and continuity of functions represented by trigonometric series: coefficient criteria*, Studia Math., **193**, 3 (2009), 285–306.
- [6] L. FENG, V. TOTIK AND S. P. ZHOU, *Trigonometric series with a generalized monotonicity condition*, Acta Math. Sin. (Engl. Ser.), **30** (2014), 1289–1296.
- [7] M. R. GABDULLIN, *Trigonometric series with noninteger harmonics*, J. Math. Anal. Appl., **508**, 1 (2022), 125792.
- [8] A. JEFFREY AND D. ZWILLINGER, *Table of Integrals, Series, and Products*, Seventh Edition, Academic Press, 2007.
- [9] P. KÓRUS, *On the uniform convergence of special sine integrals*, Acta Math. Hungar., **133** (2011), 82–91.
- [10] P. KÓRUS, *Uniform convergence of double trigonometric integrals*, Colloq. Math., **154** (2018), 107–119.
- [11] S. KESKA, *On the Uniform Convergence of Sine Series with Square Root*, J. Funct. Spaces, **2019** (2019), Article ID 1342189.
- [12] E. LIFLYAND AND S. TIKHONOV, *A concept of general monotonicity and applications*, Math. Nachr., **284**, 8–9 (2011), 1083–1098.
- [13] E. LIFLYAND AND S. TIKHONOV, *Extended solution of Boas' conjecture on Fourier transforms*, C. R. Acad. Sci. Paris, Ser. I, **346** (2008), 1137–1142.
- [14] E. LIFLYAND AND S. TIKHONOV, *The Fourier transforms of general monotone functions*, In: Analysis and Mathematical Physics, Trends in Mathematics, Birkhäuser, Basel, 2009, pp. 373–395.

- [15] E. LIFLYAND AND S. TIKHONOV, *Two-sided weighted Fourier inequalities*, Ann. Sc. Norm. Super. Pisa Cl. Sci. (5), **11**, 2 (2012), 341–362.
- [16] F. MÓRICZ, *On the uniform convergence of sine integrals*, J. Math. Anal. Appl., **354** (2009), 213–219.
- [17] K. A. OGANESYAN, *Uniform convergence criterion for non-harmonic sine series*, Mat. Sb., **212**, 1 (2021), 78–118; Sb. Math., **212**, 1 (2021), 70–110.
- [18] B. SZAL, *A new class of numerical sequences and its applications to uniform convergence of sine series*, Math. Nachr., **284**, 14–15 (2011), 1985–2002.
- [19] S. TIKHONOV, *Trigonometric series with general monotone coefficients*, J. Math. Anal. Appl., **326** (2007), 721–735.
- [20] E. C. TITCHMARSH, *Introduction to the Theory of Fourier Integrals*, second edition, Clarendon Press, Oxford, 1948.