

ON THE JACOBI–DUNKL COEFFICIENTS OF LIPSCHITZ AND DINI–LIPSCHITZ FUNCTIONS ON THE CIRCLE

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Abstract. In this paper, we consider \mathcal{E} the set of infinitely differentiable 2π -periodic functions on the circle $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$. We use the distributions in \mathcal{E} , as a tool to prove the continuity of the Jacobi–Dunkl operator. We obtain a generalization of the classical Titchmarsh theorem for the Jacobi–Dunkl coefficients of a set of functions satisfying Lipschitz conditions, with the use of the generalized Jacobi–Dunkl translation operator defined by Vinogradov. In addition, we introduce the discrete Jacobi–Dunkl Dini–Lipschitz class and we obtain an analogue of Younis’ theorem in this occurrence.

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