

## JOHN–NIRENBERG INEQUALITY FOR LIPSCHITZ MARTINGALE SPACES

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*Abstract.* In this article, John-Nirenberg inequality for Lipschitz martingale spaces is established. We further prove that Lipschitz martingale spaces  $\Lambda_p(\alpha)$  are equivalent for  $0 < p < \infty$ , which generalizes an important result in classical martingale  $H_p$  theory.

*Mathematics subject classification (2020):* 60G42, 60G46.

*Keywords and phrases:* Martingale, John-Nirenberg inequality, Lipschitz space, BMO space.

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