

## COUNTER-EXAMPLES CONCERNING BRECKNER-CONVEXITY

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**Abstract.** In this paper, we examine convexity type inequalities. Let  $D$  be a nonempty convex subset of a linear space,  $c > 0$  and  $\alpha : D - D \rightarrow \mathbb{R}$  be a given even function. The inequality

$$f\left(\frac{x+y}{2}\right) \leq cf(x) + cf(y) + \alpha(x-y) \quad (x, y \in D)$$

is the focus of our examinations. We will construct an example to show that for  $c = 1$ , this Jensen type inequality does not imply the convexity of the function. Then, we compare this inequality with Hermite–Hadamard type inequalities.

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