

APPROXIMATION BY NÖRLUND MEANS WITH RESPECT TO WALSH SYSTEM IN LEBESGUE SPACES

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Abstract. In this paper we improve and complement a result by Móricz and Siddiqi [12]. In particular, we prove that their inequality of the Nörlund means with respect to the Walsh system holds also without their additional condition. Moreover, we prove some new approximation results and inequalities in Lebesgue spaces for any $1 \leq p < \infty$.

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