ON THE ZEROS OF POLYNOMIALS AND RELATED
REGULAR FUNCTIONS OF A QUATERNIONIC VARIABLE

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Abstract. In this paper, we establish a ring-shaped region containing all the zeros of a unilateral polynomial with quaternionic coefficients located on only one side of the powers of the quaternionic variable. We shall also obtain zero-free regions for the related subclass of regular power series.

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