

LIOUVILLE TYPE THEOREMS FOR FRACTIONAL ELLIPTIC SYSTEMS WITH COUPLED TERMS

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Abstract. In this paper, we study the fractional elliptic system with coupled terms

$$\begin{cases} (-\Delta)^s u = (q+1)u^q v^{p+1} & \text{in } \mathbb{R}^N \\ (-\Delta)^s v = (p+1)v^p u^{q+1} & \text{in } \mathbb{R}^N, \end{cases}$$

where $0 < s < 1$ and $N > 2s$. We first prove that if $p > -1$, $q > -1$ and $p+q+1 \leq \frac{N}{N-2s}$, then the system has no positive supersolution. In the case $p, q > 0$ we establish the nonexistence result of stable positive solutions. Our results generalize some results in [Li, Yayun; Lei, Yutian; *Commun. Pure Appl. Anal.* 17 (2018), no. 5, 1749–1764.] to the system involving the fractional Laplacian.

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