

## SOME EXTENSIONS OF CLASSES INVOLVING PAIR OF WEIGHTS RELATED TO THE BOUNDEDNESS OF MULTILINEAR COMMUTATORS ASSOCIATED TO GENERALIZED FRACTIONAL INTEGRAL OPERATORS

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*Abstract.* We deal with the boundedness properties of higher order commutators related to some generalizations of the multilinear fractional integral operator of order  $m$ ,  $I_{\alpha}^m$ , from a product of weighted Lebesgue spaces into adequate weighted Lipschitz spaces, extending some previous estimates for the linear case. Our study includes two different types of commutators and sufficient conditions on the weights in order to guarantee the continuity properties described above. We also exhibit the optimal range of the parameters involved. The optimality is understood in the sense that the parameters defining the corresponding spaces belong to a certain region, being the weights trivial outside of it. We further show examples of weights for the class which cover the mentioned area.

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### REFERENCES

- [1] H. AIMAR, S. HARTZSTEIN, B. IAFFEI, AND B. VIVIANI, *The Riesz potential as a multilinear operator into general  $BMO_{\beta}$  spaces*, vol. 173, 2011, Problems in mathematical analysis. no. 55, pp. 643–655.
- [2] FABIO BERRA, GLADIS PRADOLINI, AND WILFREDO RAMOS, *Optimal parameters related with continuity properties of the multilinear fractional integral operator between Lebesgue and Lipschitz spaces*, Positivity **27** (2023), no. 2, Paper No. 22, 35.
- [3] FABIO BERRA, GLADIS PRADOLINI, AND WILFREDO RAMOS, *Two-weighted estimates of the multilinear fractional integral operator between weighted Lebesgue and Lipschitz spaces with optimal parameters*, Rev. Un. Mat. Argentina **66** (2023), no. 1, 69–90.
- [4] XI CHEN AND QINGYING XUE, *Weighted estimates for a class of multilinear fractional type operators*, J. Math. Anal. Appl. **362** (2010), no. 2, 355–373.
- [5] E. HARBOURE, O. SALINAS, AND B. VIVIANI, *Boundedness of the fractional integral on weighted Lebesgue and Lipschitz spaces*, Trans. Amer. Math. Soc. **349** (1997), no. 1, 235–255.
- [6] A. K. LERNER, S. OMBROSI, C. PÉREZ, R. H. TORRES, AND R. TRUJILLO-GONZÁLEZ, *New maximal functions and multiple weights for the multilinear Calderón-Zygmund theory*, Adv. Math. **220** (2009), no. 4, 1222–1264.
- [7] KABE MOEN, *Weighted inequalities for multilinear fractional integral operators*, Collect. Math. **60** (2009), no. 2, 213–238.
- [8] B. MUCKENHOUPT AND R. WHEEDEN, *Weighted norm inequalities for fractional integrals*, Trans. Amer. Math. Soc. **192** (1974), 261–274.
- [9] JAAK PEETRE, *On the theory of  $\mathcal{L}_{p,\lambda}$  spaces*, J. Functional Analysis **4** (1969), 71–87.
- [10] CARLOS PÉREZ, *Two weighted inequalities for potential and fractional type maximal operators*, Indiana Univ. Math. J. **43** (1994), no. 2, 663–683.

- [11] CARLOS PÉREZ, GLADIS PRADOLINI, RODOLFO H. TORRES, AND RODRIGO TRUJILLO-GONZÁLEZ, *End-point estimates for iterated commutators of multilinear singular integrals*, Bull. Lond. Math. Soc. **46** (2014), no. 1, 26–42.
- [12] CARLOS PÉREZ AND RODOLFO H. TORRES, *Sharp maximal function estimates for multilinear singular integrals*, Harmonic analysis at Mount Holyoke (South Hadley, MA, 2001), Contemp. Math., vol. 320, Amer. Math. Soc., Providence, RI, 2003, pp. 323–331.
- [13] GLADIS PRADOLINI, *A class of pairs of weights related to the boundedness of the fractional integral operator between  $L^p$  and Lipschitz spaces*, Comment. Math. Univ. Carolin. **42** (2001), no. 1, 133–152.
- [14] GLADIS PRADOLINI, *Two-weighted norm inequalities for the fractional integral operator between  $L^p$  and Lipschitz spaces*, Comment. Math. (Prace Mat.) **41** (2001), 147–169.
- [15] GLADIS PRADOLINI, WILFREDO RAMOS, AND JORGELINA RECCHI, *On the optimal numerical parameters related with two weighted estimates for commutators of classical operators and extrapolation results*, Collect. Math. **72** (2021), no. 1, 229–259.
- [16] GLADIS PRADOLINI AND JORGELINA RECCHI, *On optimal parameters involved with two-weighted estimates of commutators of singular and fractional operators with Lipschitz symbols*, Czechoslovak Math. J. **73** (148) (2023), no. 3, 733–754.
- [17] E. SAWYER AND R. L. WHEEDEN, *Weighted inequalities for fractional integrals on Euclidean and homogeneous spaces*, Amer. J. Math. **114** (1992), no. 4, 813–874.