

ON A GENERALIZED JORDAN–VON NEUMANN TYPE CONSTANT AND NORMAL STRUCTURE

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Abstract. In this paper, we introduce a new geometric constant $C_{-\infty}^{(p)}(a, X)$, which is closely related to the generalized Jordan–von Neumann type constant. We show that 2 and $\frac{(a+2)^p}{2^{p-2}(2^p+a^p)}$ are the upper and lower bound for $C_{-\infty}^{(p)}(a, X)$, respectively. Moreover, we obtain that $C_{-\infty}^{(p)}(a, X) = C_{-\infty}^{(p)}(a, \tilde{X})$, where \tilde{X} is the ultrapower space of X . Subsequently, we give some sufficient conditions for normal structure of a Banach space with different constants, such as the generalized James constant, Domínguez-Benavides coefficient and the coefficient of weak orthogonality.

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