

SOME GEOMETRIC CONSTANTS RELATED TO ρ -ORTHOGONALITY AND ρ' -ORTHOGONALITY IN BANACH SPACES

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Abstract. In this article, we introduce two new moduli of convexity $\delta_{\perp}(\rho, X)$ and $\delta_{\perp}(\rho', X)$ related to ρ -orthogonality and ρ' -orthogonality, which are connected with the modulus of convexity $\delta_X(\varepsilon)$. The connections between these two parameters and other well-known constants are built. In the meantime, this two new coefficients are calculated for X being some specific spaces. Moreover, we also provide a characterization of the Radon plane with affine-regular hexagonal unit sphere in terms of $\delta_{\perp}(\rho, X)$. To consider the moduli of smoothness related to ρ -orthogonality and ρ' -orthogonality, we also treat $\rho_{\perp}(\rho, X)$ and $\rho_{\perp}(\rho', X)$.

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