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MOVING AROUND THE SUMS OF ORTHOGONAL UNIT VECTORS

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Abstract. We discuss the extreme values of the sum of two Birkhoff orthogonal unit vectors in a normed space. In addition, we obtain some relationships between these values with some moduli of convexity and smoothness, as well as with the notions of uniform convexity or uniform non-squareness. Finally, we present some illustrative examples.

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