ON THE NUMERICAL RADIUS OF AN OPERATOR MATRIX

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Abstract. This paper investigates possible upper and lower bounds for the numerical radii of 2×2 operator matrices.

The obtained results improve and generalize many results from the literature in accessible forms. Moreover, many numerical examples will be given to support the feasibility of our findings.

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