Φ–MOMENT B–VALUED MARTINGALE
INEQUALITIES ON LORENTZ SPACES

LIBO LI*, KAITUO LIU, LIN WANG AND LIN YU

Abstract. In this paper, with the help of some new atomic decomposition theorems, several Φ-moment Banach space valued martingale inequalities associated with concave functions in the context of Lorentz spaces are deduced. Our results are closely related with the geometrical properties of the underlying Banach spaces.

Mathematics subject classification (2020): 60G46, 60G42.

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REFERENCES

[1] C. BENNETT AND R. SHARPLEY, *Interpolation of Operators*, Pure and Applied Mathematics, **129**, Academic Press, Inc. Boston, MA, 1988.

[2] A. BERNARD AND B. MAISONNEUVE, *Décomposition atomique de martingales de la classe H^1*, Lecture Notes in Math. **581**, pp. 303–326, Springer, Berlin-New York, 1977.

[3] D. L. BURKHOLDER, *Martingale transforms and the geometry of Banach spaces*, Proceedings of the Third International Conference on Probability in Banach Spaces (Tufts University 1980), Lecture Notes in Math. **860**, pp. 35–50, Springer, Berlin-New York, 1981.

[4] D. L. BURKHOLDER, *Differential subordination of harmonic functions and martingales*, Harmonic Analysis and Partial Differential Equations (El Escorial 1987), Lecture Notes in Math. **1384**, pp. 1–23, Springer, Berlin, 1989.

[5] D. L. BURKHOLDER, B. J. DAVIS AND R. F. GUNDY, *Integral inequalities for convex functions of operators on martingales*, Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability (Univ. California, Berkeley, Calif. 1970/1971), Vol. II: Probability theory, pp. 223–240, Univ. California Press, Berkeley, CA, 1972.

[6] D. L. BURKHOLDER AND R. F. GUNDY, *Extrapolation and interpolation of quasi-linear operators on martingales*, Acta Math. **124** (1970), 249–304.

[7] L. GRAFAKOS, *Classical Fourier analysis*, Third edition. Graduate Texts in Mathematics, **249**, Springer, New York, 2014.

[8] Z. HAO, L. LI AND A. YANG, Φ-*moment martingale inequalities on Lorentz spaces with variable exponents*, Banach J. Math. Anal. **17** (2023), no. 1, Paper No. 12, 26 pp.

[9] C. HERZ, *H_p-spaces of martingales, $0 < p \leqslant 1$*, Z. Wahrscheinlichkeitstheorie und Verw. Gebiete **28** (1973/74), 189–205.

[10] J. HOFFMANN-JØRGENSEN AND G. PISIER, *The law of large numbers and the central limit theorem in Banach spaces*, Ann. Probab. **4** (1976), no. 4, 587–599.

[11] Y. HOU AND Y. REN, *Weak martingale Hardy spaces and weak atomic decompositions*, Sci. China Ser. A **49** (2006), no. 7, 912–921.

[12] Y. JIAO, L. PENG AND P. LIU, *Atomic decompositions of Lorentz martingale spaces and applications*, J. Funct. Spaces Appl. **7** (2009), no. 2, 153–166.

[13] Y. JIAO, L. WU AND L. PENG, *Weak Orlicz-Hardy martingale spaces*, Internat. J. Math. **26** (2015), no. 8, 1550062, 26 pp.

[14] Y. JIAO AND W. YU, *Some new Φ-moment martingale inequalities associated with concave functions*, Nonlinear Anal. **75** (2012), no. 11, 4271–4277.

[15] M. KIKUCHI, *On some inequalities for Doob decompositions in Banach function spaces*, Math. Z. **265** (2010), no. 4, 865–887.

[16] Y. LI AND P. LIU, *Weak atomic decompositions of B-valued martingales with two-parameters*, Acta Math. Hungar. **127** (2010), no. 3, 225–238.

[17] K. LIU, W. LI AND T. YUE, *B-valued martingale Hardy-Lorentz-Karamata spaces*, Bull. Malays. Math. Sci. Soc. **42** (2019), no. 5, 2395–2422.

[18] K. LIU, D. ZHOU AND Y. JIAO, *Hardy-Lorentz spaces for B-valued martingales*, J. Math. Anal. Appl. **450** (2017), no. 2, 1401–1420.

[19] P. LIU, *Martingale and geometry of Banach spaces*, Science Press, Beijing, 2007.

[20] P. LIU AND Y. HOU, *Atomic decompositions of Banach-space-valued martingales*, Sci. China Ser. A **42** (1999), no. 1, 38–47.

[21] P. LIU AND L. YU, *B-valued martingale spaces with small index and atomic decompositions*, Sci. China Ser. A **44** (2001), no. 11, 1361–1372.

[22] R. LONG, *Martingale spaces and inequalities*, Peking University Press, Beijing; Friedr. Vieweg-Sohn, Braunschweig, 1993.

[23] G. G. LORENTZ, *Some new functional spaces*, Ann. of Math. **51** (1950), no. 1, 37–55.

[24] Y. S. MISHURA AND F. WEISZ, *Atomic decompositions and inequalities for vector-valued discrete-time martingales*, Theory Probab. Appl. **43** (1999), no. 3, 487–496.

[25] Y. S. MISHURA AND F. WEISZ, *Inequalities for vector-valued martingales with continuous time*, Theory Prob. Math. Statist. **58** (1999), 9–25.

[26] T. MIYAMOTO, E. NAKAI AND G. SADASUE, *Martingale Orlicz-Hardy spaces*, Math. Nachr. **285** (2012), no. 5–6, 670–686.

[27] L. PENG AND J. LI, *A generalization of Φ-moment martingale inequalities*, Stat. Probab. Lett. **102** (2015), 61–68.

[28] G. PISIER, *Martingales with values in uniformly convex spaces*, Israel J. Math. **20** (1975), no. 3–4, 326–350.

[29] G. PISIER, *Martingales in Banach spaces*, Cambridge Studies in Advanced Mathematics, vol. **155**, Cambridge University Press, Cambridge, 2016.

[30] F. WEISZ, *Martingale Hardy spaces for $0 < p \leqslant 1$*, Probab. Theory Relat. Fields **84** (1990), no. 3, 361–376.

[31] F. WEISZ, *Martingale Hardy spaces and their applications in Fourier analysis*, Lecture Notes in Mathematics, **1568**, Springer-Verlag, Berlin, 1994.

[32] L. YU, *Duality theorem for B-valued martingale Orlicz-Hardy spaces associated with concave functions*, Math. Nachr. **289** (2016), no. 5–6, 756–774.

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