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SUCCESSIVE COEFFICIENTS AND TOEPLITZ DETERMINANT FOR CONCAVE UNIVALENT FUNCTIONS

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Abstract. Let $Co(p)$ be the class of all functions f defined in the unit disc $\mathbb{D}$ having a simple pole at $z=p$ where $0<p<1$ and analytic in $\mathbb{D}\setminus\{p\}$ with $f(0)=0=f'(0)-1$ such that f maps $\mathbb{D}$ onto a domain whose complement with respect to the extended complex plane is a bounded convex set. These functions are called concave univalent functions. Each $f\in Co(p)$ has the following Taylor expansion:

$$f(z)=z+\sum_{n=2}^{\infty}a_n z^n,\quad |z|<p.$$

In this article, we first determine the regions of variability of the difference of successive coefficients $(a_{n+1}-a_n)$ for $n\geqslant 3$. We also find sharp upper bounds of the Toeplitz determinants, the entries of which are the Taylor coefficients of functions in $Co(p)$.

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Paper MIA-27-31

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