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JENSEN–TYPE INEQUALITIES IN TERMS OF LIPSCHITZIANITY

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Abstract. The main focus of this paper is a study of Jensen-type inequalities for the Lipschitzian functions. We establish the reverse of the Jensen inequality expressed in terms of the corresponding Lipschitz constant. In addition, we also obtain the reverse of the superadditivity relation for a convex function, expressed in the same way. As an application, we obtain reverses of power mean inequalities, the Hölder inequality, and the Hermite-Hadamard inequality, expressed in terms of the Lipschitzianity. In particular, we derive reverses of the arithmetic-geometric mean inequality in both difference and quotient forms.

Mathematics subject classification (2020): 26A51, 26A16, 26D15.
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REFERENCES

[1] C. CONDE, N. MINCULETE, H. R. MORADI, M. SABABHEH, *Norm inequalities via convex and log-convex functions*, Mediterr. J. Math. **20**, 6 (2023), https://doi.org/10.1007/s00009-022-02214-z
[2] S. S. DRAGOMIR, *Bounds for the normalised Jensen functional*, Bull. Austral. Math. Soc. **3** (2006), 471–478.
[3] G. KARAMALI, H. R. MORADI, M. SABABHEH, *More about operator order preserving*, Rocky Mountain J. Math. **51** (5) (2021), 1691–1699.
[4] V. N. KONOVALOV, K. A. KOPOTUN, V. E. MAIOROV, *Convex Polynomial and Ridge Approximation of Lipschitz Functions in* $\mathbb{R}^d$, Rocky Mountain J. Math. **40** (3) (2010), 957–976.
[5] M. KRNIĆ, N. LOVRIČEVIĆ, J. PEČARIĆ, *On the properties of McShane's functional and their applications*, Period. Math. Hung. **66** (2013), 159–180.
[6] M. KRNIĆ, N. LOVRIČEVIĆ, J. PEČARIĆ, *Jessen's functional, its properties and applications*, An. Şt. Univ. Ovidius Constanţa. **20** (2012), 225–248.
[7] D. S. MITRINOVIĆ, J. E. PEČARIĆ, A. M. FINK, *Classical and New Inequalities in Analysis*, Kluwer Academic Publishers, Dordrecht/Boston/London, 1993.
[8] H. R. MORADI, S. FURUICHI, M. SABABHEH, *Some operator inequalities via convexity*, Linear Multilinear Algebra. **70** (2022), 7740–7752, https://doi.org/10.1080/03081087.2021.2006592.
[9] H. R. MORADI, M. SABABHEH, S. FURUICHI, *On the operator Hermite-Hadamard inequality*, Complex Anal. Oper. Theory **15**, 122 (2021), https://doi.org/10.1007/s11785-021-01172-w.
[10] L. NASIRI, A. ZARDADI, H. R. MORADI, *Refining and reversing Jensen's inequality*, Oper. Matrices. **16** (1) (2022), 19–27.
[11] K. PASTOR, *Characterization of Strict Convexity for Locally Lipschitz Functions*, Rocky Mountain J. Math. **39** (6) (2009), 2029–2049.
[12] J. E. PEČARIĆ, F. PROSCHAN, Y. L. TONG, *Convex Functions, Partial Orderings, and Statistical Applications*, Academic Press, Inc, 1992.
[13] A. W. ROBERTS, D. E. VARBEG, *Convex Functions*, Academic Press, New York and London (1973).
[14] M. SABABHEH, S. FURUICHI, H. R. MORADI, *Improved matrix inequalities using radical convexity*, J. Math. Inequal. **17** (1) (2023), 349–364.

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[15] M. SABABHEH, S. FURUICHI, H. R. MORADI, *Composite convex functions*, J. Math. Inequal. **15** (3) (2021), 1267–1285.
[16] M. SABABHEH, H. R. MORADI, I. H. GÜMÜŞ, *Further inequalities for convex functions*, Bull. Malays. Math. Sci. Soc. **46**, 42 (2023), https://doi.org/10.1007/s40840-022-01438-2.
[17] M. SABABHEH, H. R. MORADI, *Radical convex functions*, Mediterr. J. Math. **18**, 137 (2021), https://doi.org/10.1007/s00009-021-01784-8.
[18] V. TUAN, C. TAMMER, C. ZALINESCU, *The Lipschitzianity of convex vector and set-valued functions*, Top **24** (2016), 279–299.