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CURVATURE ESTIMATES OF A SPACELIKE GRAPH IN A LORENTZIAN PRODUCT SPACE

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Abstract. Let M be an n-dimensional complete Riemannian manifold with the metric $\langle,\rangle_M$ and let $M\times\mathbb{R}_1$ be a Lorentzian product space $M\times\mathbb{R}$ with the metric $\langle,\rangle_M - dt^2$. We first provide Heinz type curvature estimates for the spacelike graph in $M\times\mathbb{R}_1$ of a C^2-function f defined on a closed geodesic ball $\overline{B}_{x_0}(R)$ of radius R centered at x_0 on M. In particular, the estimates are related to the radius R and the value of $\|\nabla f(x_1)\|$ for which $f(x_1)=\max_{\partial\overline{B}_{x_0}(R)} f$. Secondly, we give L^2-estimates of the mean curvature for a spacelike graph defined on a compact Riemannian manifold.

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