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ON THE INVARIANCE EQUATION FOR MEANS OF GENERALIZED POWER GROWTH

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Abstract. We generalize the result of (Witkowski, 2014) which binds orders of homogeneous, symmetric means $M,N,K\colon \mathbb{R}_+^2 \to \mathbb{R}_+$ of power growth that satisfy the invariance equation $K(M(x,y),N(x,y)) = K(x,y)$ to the broader class of means.

Moreover, we define the lower– and the upper–order which gives us insight into the order of the solution of this equation in the case when means do not belong to this class.

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