

A NEW ESTIMATE OF THE TRANSFINITE DIAMETER OF BERNSTEIN SETS

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Abstract. Let $K \subset \mathbb{C}^n$ be a compact set satisfying the following Bernstein inequality: for any $m \in \{1, \dots, n\}$ and for any n -variate polynomial P of degree $\deg(P)$ we have

$$\max_{z \in K} \left| \frac{\partial P}{\partial z_m}(z) \right| \leq M \deg(P) \max_{z \in K} |P(z)| \quad \text{for } z = (z_1, \dots, z_n)$$

for some constant $M = M(K) > 0$ depending only on K . We show that the transfinite diameter of K , denoted $\delta(K)$, verifies the following lower estimate

$$\delta(K) \geq \frac{1}{nM},$$

which is optimal in the one-dimensional case. In addition, we show that if K is a Cartesian product of compact planar sets then

$$\delta(K) \geq \frac{1}{M}.$$

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