

## FURTHER GENERALIZATIONS OF ALZER–FONSECA–KOVAČEC TYPE INEQUALITIES AND APPLICATIONS

TRAN DINH PHUNG AND DUONG QUOC HUY\*

*Abstract.* In this paper, we develop a new method which allows us to establish interesting generalizations of the well-known Young-type inequalities, or inequalities between arithmetic and harmonic mean. Several attractive applications of these inequalities to matrix inequalities, determinant inequalities and unitarily invariant norm inequalities are also presented.

*Mathematics subject classification (2020):* 15A45, 47A30, 15A60, 26D07.

*Keywords and phrases:* Young inequality, trace inequality, norm inequality, operator inequality, determinant inequality.

### REFERENCES

- [1] H. ALZER, C. M. FONSECA AND A. KOVAČEC, *Young-type inequalities and their matrix analogues*, Linear Multilinear Algebra **63**, 3 (2015), 622–635.
- [2] D. CHOI, *A generalization of Young-type inequalities*, Math. Inequal. Appl. **21**, 1 (2018), 99–106.
- [3] S. S. DRAGOMIR, *Bounds for the normalised Jensen functional*, Bull. Austral. Math. Soc. **74**, 3 (2006), 471–478.
- [4] R. HORN AND C. R. JOHNSON, *Matrix Analysis*, Cambridge UP, 2013.
- [5] D. Q. HUY, D. T. T. VAN AND D. T. XINH, *Some generalizations of real power form for Young-type inequalities and their applications*, Linear Algebra App. **656**, 1 (2023), 368–384.
- [6] F. KITTANEH AND Y. MANASRAH, *Improved Young and Heinz inequalities for matrices*, J. Math. Anal. Appl. **361**, 1 (2010), 262–269.
- [7] F. KITTANEH AND Y. MANASRAH, *Reverse Young and Heinz inequalities for matrices*, Linear Multilinear Algebra **59**, 9 (2011), 1031–1037.
- [8] D. S. MITRINOVIĆ, J. E. PEČARIĆ AND A. M. FINK, *Classical and new inequalities in analysis*, Math. Appl. (East European Ser.), **61** Kluwer Academic Publishers Group, Dordrecht, 1993.
- [9] Y. REN, *Some results of Young-type inequalities*, Rev. R. Acad. Cienc. Exactas Fis. Nat. Ser. A Mat. RACSAM **114**, 143 (2020), 10.
- [10] Y. REN, *A generalized refinement of Young’s inequality*, J. Math. Inequal. **17**, 4 (2023), 1463–1470.
- [11] C. YANG AND Z. WANG, *Some new improvements of Young’s inequalities*, J. Math. Inequal. **17**, 1 (2023), 315–328.
- [12] X. YANG, C. YANG AND H. LI, *Further improvements for Young’s inequalities on the Arithmetic, Geometric, and Harmonic mean*, J. Math. Inequal. **18**, 1 (2024), 315–328.