

## CESÀRO-LIKE OPERATOR ACTING BETWEEN BLOCH TYPE SPACES

PENGCHENG TANG\* AND XUEJUN ZHANG

*Abstract.* Let  $\mu$  be a finite positive Borel measure on the interval  $[0, 1)$  and  $f(z) = \sum_{n=0}^{\infty} a_n z^n \in H(\mathbb{D})$ . The Cesàro-like operator is defined by

$$\mathcal{C}_{\mu}(f)(z) = \sum_{n=0}^{\infty} \left( \mu_n \sum_{k=0}^n a_k \right) z^n, \quad z \in \mathbb{D},$$

where, for  $n \geq 0$ ,  $\mu_n$  denotes the  $n$ -th moment of the measure  $\mu$ , that is,  $\mu_n = \int_{[0,1)} t^n d\mu(t)$ .

In this paper we investigate the action of the operators  $\mathcal{C}_{\mu}$  from one Bloch type spaces  $\mathcal{B}^{\alpha}$  into another one  $\mathcal{B}^{\beta}$ .

*Mathematics subject classification (2020):* 47B38, 30H30.

*Keywords and phrases:* Cesàro-like operator, Carleson measure, Bloch-type spaces.

### REFERENCES

- [1] A. ALEMAN AND A. SISKAKIS, *Integration operators on Bergman spaces*, Indiana Univ. Math. J. **46** 2 (1997), 337–356.
- [2] A. ALEMAN AND J. CIMA, *An integral operator on  $H^p$  and Hardy's inequality*, J. Anal. Math. **85** (2001), 157–176.
- [3] G. BAO, F. SUN AND H. WULAN, *Carleson measure and the range of Cesàro-like operator acting on  $H^{\infty}$* , Anal. Math. Phys. **12** (2022), no. 142.
- [4] G. BAO AND H. WULAN, *Hankel matrices acting on Dirichlet spaces*, J. Math. Anal. Appl. **409** (2014), 228–235.
- [5] G. BAO, H. WULAN AND F. YE, *The range of the Cesàro operator acting on  $H^{\infty}$* , Canad. Math. Bull. **31** (2020), 633–642.
- [6] G. BAO, K. GUO, F. SUN AND Z. WANG, *Hankel matrices acting on the Dirichlet space*, J. Fourier Anal. Appl. **30** (2024), no. 53.
- [7] M. BELTRÁN-MENEU, J. BONET AND E. JORDÁ, *Cesàro operators associated with Borel measures acting on weighted spaces of holomorphic functions with sup-norms*, Anal. Math. Phys. **14** (2024), no. 109.
- [8] O. BLASCO, *Cesàro-type operators on Hardy spaces*, J. Math. Anal. Appl. **529** (2) (2023), no. 127017.
- [9] O. BLASCO, *Generalized Cesàro operators on weighted Dirichlet spaces*, J. Math. Anal. Appl. **540** (1) (2024), no. 128627.
- [10] C. CHANG, S. LI AND S. STEVIĆ, *On some integral operators on the unit polydisk and the unit ball*, Taiwanese J. Math. **11** (5) (2007), 1251–1285.
- [11] C. CHANG AND S. STEVIĆ, *The generalized Cesàro operator on the unit polydisk*, Taiwanese J. Math. **7** (2) (2003), 293–308.
- [12] N. DANIKAS AND A. SISKAKIS, *The Cesàro operator on bounded analytic functions*, Analysis **13** (1993), 295–299.
- [13] P. DUREN, *Theory of  $H^p$  spaces*, Academic Press, New York, 1970.
- [14] P. GALANOPOULOS, *The Cesàro operator on Dirichlet spaces*, Acta Sci. Math.(Szeged) **67** (2001), 411–420.
- [15] P. GALANOPOULOS, D. GIRELA, A. MAS AND N. MERCHÁN, *Operators induced by radial measures acting on the Dirichlet space*, Results Math. **78** (2023), no. 106.

- [16] P. GALANOPOULOS, D. GIRELA AND N. MERCHÁN, *Cesàro-like operators acting on spaces of analytic functions*, Anal. Math. Phys. **12** (2022), no. 51.
- [17] P. GALANOPOULOS, D. GIRELA AND N. MERCHÁN, *Cesàro-type operators associated with Borel measures on the unit disc acting on some Hilbert spaces of analytic functions*, J. Math. Anal. Appl. **526** (2023), no. 127287.
- [18] P. GALANOPOULOS AND J. PELÁEZ, *A Hankel matrix acting on Hardy and Bergman spaces*, Studia Math. **200** (3) (2010), 201–220.
- [19] Y. GUO, P. TANG AND X. ZHANG, *Cesàro-like operators between the Bloch space and Bergman spaces*, Ann. Funct. Anal. **15** (2024), no. 8.
- [20] Z. HU, *Extended Cesàro operators on the Bloch space in the unit ball of  $C^n$* , Acta Math. Sci. **23** (B) (2003), 561–566.
- [21] J. JIN AND S. TANG, *Generalized Cesàro operator on Dirichlet-type spaces*, Acta Math. Sci. **42**(B) (2022), 212–220.
- [22] I. KAYUMOV AND K. WIRTHS, *Coefficients problems for Bloch functions*, Anal. Math. Phys. **9** (3) (2019), 1069–1085.
- [23] S. LI AND S. STEVIĆ, *Products of integral-type operators and composition operators between Bloch-type spaces*, J. Math. Anal. Appl. **349** (2009), 596–610.
- [24] M. LINDSTRÖM, D. NORRBO AND S. STEVIĆ, *On compactness of operators from Banach spaces of holomorphic functions to Banach spaces*, J. Math. Inequal. **18** (3) (2024), 1153–1158.
- [25] J. MIAO, *The Cesàro operator is bounded on  $H^p$  for  $0 < p < 1$* , Proc. Amer. Math. Soc. **116** (1992), 1077–1079.
- [26] V. MILLER AND T. MILLER, *The Cesàro operator on the Bergman space  $A^2(\mathbb{D})$* , Arch. Math. **78** (2002), 409–416.
- [27] J. PELÁEZ AND J. RÄTTYÄ, *Weighted Bergman spaces induced by rapidly increasing weights*, Mem. Amer. Math. Soc. **227** (2014), 1–122.
- [28] A. SISKAKIS, *On the Bergman space norm of the Cesàro operator*, Arch. Math. **67** (1996), 4312–318.
- [29] A. SISKAKIS, *Composition semigroups and the Cesàro operator on  $H^p$* , J. London Math. Soc. **36** (1987), 153–164.
- [30] A. SISKAKIS, *The Cesàro operator is bounded on  $H^1$* , Proc. Amer. Math. Soc. **110** (1990), 461–462.
- [31] S. STEVIĆ, *Boundedness and compactness of an integral operator on a weighted space on the poly-disc*, Indian J. Pure Appl. Math. **37** (6) (2006), 343–355.
- [32] S. STEVIĆ, *Cesàro averaging operators*, Math. Nachr. **248–249** (2003), 185–189.
- [33] S. STEVIĆ, *Integral-type operators from a mixed norm space to a Bloch-type space on the unit ball*, Siberian Math. J. **50** (6) (2009), 1098–1105.
- [34] S. STEVIĆ, *On an integral operator between Bloch-type spaces on the unit ball*, Bull. Sci. Math. **134** (2010), 329–339.
- [35] S. STEVIĆ AND S. UEKI, *Integral-type operators acting between weighted-type spaces on the ball*, Appl. Math. Comput. **215** (7) (2009), 2464–2471.
- [36] F. SUN, F. YE AND L. ZHOU, *A Cesàro-like operator from Besov spaces to some spaces of analytic functions*, Comput. Methods Funct. Theory (2024), <https://doi.org/10.1007/s40315-024-00542-7>.
- [37] P. TANG AND X. ZHANG, *Generalized integral type Hilbert operator acting on weighted Bloch spaces*, Math. Meth. Appl. Sci. **46** (2023) 18458–18472.
- [38] H. WULAN AND K. ZHU, *Möbius invariant  $Q_K$  spaces*, Berlin: Springer-Verlag, 2017.
- [39] J. XIAO, *Cesàro-type operators on Hardy, BMOA and Bloch spaces*, Arch. Math. **68** (1997) 398–406.
- [40] X. ZHANG, Y. GUO, Q. SHANG AND S. LI, *The Gleason's problem on  $F(p, q, s)$  type spaces in the unit ball of  $C^n$* , Complex Anal. Oper. Theory **12** (2018), 1251–1265.
- [41] R. ZHAO, *On logarithmic Carleson measures*, Acta Sci. Math. (Szeged) **69** (3–4) (2003), 605–618.
- [42] Z. ZHOU, *Pseudo-Carleson measures and generalized Cesàro-like operators*, preprint.
- [43] K. ZHU, *Spaces of holomorphic functions in the unit ball*, Springer-Verlag (GTM 226), New York, 2005.