

## A PROBABILITY ARGUMENT IN FAVOR OF IGNORING SMALL SINGULAR VALUES

ALBRECHT BÖTTCHER, DANIEL POTTS AND DAVID WENZEL

*Abstract.* If the matrix of a square linear system is nonsingular but has very small singular values, then tiny perturbations of the right-hand side may cause drastic changes in the solution. We show that the probability for this to happen is very close to zero if sufficiently many singular values of the matrix are bounded away from zero.

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