

WEAK CONTRACTIONS AND TRACE CLASS PERTURBATIONS

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Abstract. An absolutely continuous contraction is said to be in the class \mathbb{A} if it has isometric H^∞ functional calculus. We present evidence in favor of the conjecture that the class \mathbb{A} is invariant under trace-class perturbations.

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