

LOCALIZATIONS OF THE KLEINECKE–SHIROKOV THEOREM

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Abstract. A local version of the Kleinecke-Shirokov theorem is proved. The results easily extend to bounded linear derivations on Banach algebras. In case of algebraic elements, an improved bound on the nilindex of a commutator is obtained as a consequence.

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