

## APPROXIMATE PERMUTABILITY OF TRACES ON SEMIGROUPS OF MATRICES

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*Abstract.* It is known that if trace is permutable on a semigroup  $\mathcal{S}$  of complex matrices, i.e.,  $\text{tr}(ABC) = \text{tr}(BAC)$  for all  $A, B, C$  in  $\mathcal{S}$ , then  $\mathcal{S}$  is triangularizable. We study an approximate version of this condition:  $|\text{tr}(ABC - BAC)| \leq \varepsilon \rho(A)\rho(B)\rho(C)$  for all  $A, B, C$  in  $\mathcal{S}$ , where  $\rho$  is the spectral radius. We show that this condition with  $\varepsilon < 3$  yields commutativity for compact groups and triangularizability for certain groups including connected ones. For general semigroups additional assumptions are needed. Moreover, we show that any property on semigroups of matrices that satisfies certain pretriangularizing conditions, yields similar conclusions.

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