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A CLASS OF TRIDIAGONAL REPRODUCING KERNELS

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Abstract. The class of analytic reproducing kernels

$$K_p(z,w) = \sum_{n=0}^{\infty} f_n(z)\overline{f_n(w)}$$

is considered where $f_n(z) = (1 - b_n z)z^n$ with $b_n = (\frac{n+1}{n+2})^p$ and $p > 0$. In this case $H(K_p)$ consists of functions with domain $\mathbb{D} \cup \{1\}$. For each p, a concrete realization of $H(K_p)$ is provided. For the case $p > 1/2$, $H(K_p)$ is shown to have the factorization property and the operator of multiplication by z is shown to be similar to a rank one perturbation of the unilateral shift. A characterization of the multiplier algebra of $H(K_p)$ is given for all values of $p > 0$.

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