

CONVERGENCE AND DECOMPOSITION FOR TENSOR PRODUCTS OF HILBERT SPACE OPERATORS

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Abstract. It is shown that convergence of sequences of Hilbert space operators is preserved by tensor product and the converse holds in case of convergence to zero under the semigroup assumption. In particular, unlike ordinary product of operators, weak convergence is preserved by tensor product. It is also shown that a tensor product of operators is a unilateral shift if and only if it coincides with a tensor product of a unilateral shift and an isometry. These results lead to a decomposition of a tensor product of contractions into an orthogonal direct sum of tensor products of class \mathcal{C}_{00} , strongly stable tensor products, unilateral shift tensor products, and a unitary tensor product.

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