

## PROOF OF BÖTTCHER AND WENZEL'S CONJECTURE

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*Abstract.* In 2005, Böttcher and Wenzel raised a conjecture that if  $X$  and  $Y$  are any two real  $n$ -by- $n$  matrices, then  $\|XY - YX\|_F^2 \leq 2\|X\|_F^2\|Y\|_F^2$  where  $\|\cdot\|_F$  denotes the Frobenius norm. They proved this for the case of 2-by-2 matrices. Later, László proved the conjecture for the case of 3-by-3 matrices. In this paper, we prove the conjecture for general  $n$ -by- $n$  matrices.

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*Key words and phrases:* Commutator of two matrices, the Cauchy-Schwarz inequality, the Lagrange identity.

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