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BISHOP'S PROPERTY (β) FOR PARANORMAL OPERATORS

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Abstract. For an operator T on a separable complex Hilbert space $\mathscr{H}$, we say that T has Bishop's property (β) if for any open subset $\mathscr{D} \subset \mathbb{C}$ and any sequence of analytic functions $f_n : \mathscr{D} \to \mathscr{H}$ such as $\|(T-z)f_n(z)\| \to 0$ as $n \to \infty$ uniformly on every compact subset $\mathscr{K} \subset \mathscr{D}$, then $f_n \to 0$ uniformly on $\mathscr{K}$. It is a very important property in spectral theory. It is well-known that every normal operator ($T^*T = TT^*$) has Bishop's property (β). Now, many mathematicians attempt to extend this result to non-normal operators.

In this paper, we shall show that every paranormal operator ($\|T^2x\|\|x\| \geqslant \|Tx\|^2$ for all $x \in \mathscr{H}$) has Bishop's property (β).

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