

LINEAR MAPS PRESERVING THE MINIMUM MODULUS

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Abstract. We characterize surjective linear maps that preserve the minimum modulus between unital semisimple Banach algebras, one of them is a unital C^* -algebra having either real rank zero or essential socle. We also describe surjective linear maps on $\mathcal{L}(H)$, with H an infinite-dimensional Hilbert space, preserving the essential minimum modulus. Results concerning surjectivity and maximum modulus are also obtained.

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