

SUFFICIENT CONDITIONS FOR COMPLETE POSITIVITY

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Abstract. Marcus and Minc gave sufficient conditions on the diagonal entries of a doubly nonnegative doubly stochastic $n \times n$ matrix A , that there is a doubly nonnegative doubly stochastic matrix C with $A = C^2$. In this event, A is completely positive. We shall assume that A is doubly nonnegative and irreducible and provide slightly more general sufficient conditions on the diagonal entries of A for the existence of C . Our main result provides sufficient conditions on the principal 2×2 minors of a doubly nonnegative doubly stochastic irreducible matrix A for the existence of C .

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