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THE GENERAL SOLUTION TO A SYSTEM OF ADJOINTABLE OPERATOR EQUATIONS OVER HILBERT C^*–MODULES

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Abstract. We establish necessary and sufficient conditions for the existence of solution to the system of adjointable operator equations $A_1X = D_1, XB_2 = D_2, A_3XB_3 + B_3^*X^*C_3 = D_3$ over the Hilbert C^*-modules. We also give the explicit expression of the general solution to this system when the solvability conditions are satisfied. As an application, we investigate the anti-reflexive Hermitian solution to the system of complex matrix equations $AX = B, XC = D, EXE^* = F$. The findings of this paper extend some known results in the literature.

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