

## THE BOURGAIN–TZAFRIRI CONJECTURE AND CONCRETE CONSTRUCTIONS OF NON–PAVABLE PROJECTIONS

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*Abstract.* It is known that the Kadison-Singer Problem (KS) and the Paving Conjecture (PC) are equivalent to the Bourgain-Tzafriri Conjecture (BT). Also, it is known that (PC) fails for 2-paving projections with constant diagonal  $1/2$ . But the proofs of this fact are existence proofs. We will use variations of the discrete Fourier Transform matrices to construct concrete examples of these projections and projections with constant diagonal  $1/r$  which are not  $r$ -pavable in a very strong sense.

In 1989, Bourgain and Tzafriri showed that the class of zero diagonal matrices with small entries (on the order of  $\leq 1/\log^{1+\varepsilon} n$ , for an  $n$ -dimensional Hilbert space) are *pavable*. It has always been assumed that this result also holds for the BT-Conjecture – although no one formally checked it. We will show that this is not the case. We will show that if the BT-Conjecture is true for vectors with small coefficients (on the order of  $\leq C/\sqrt{n}$ ) then the BT-Conjecture is true and hence KS and PC are true.

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