

EIGENVALUE PROBLEM MEETS SIERPINSKI TRIANGLE: COMPUTING THE SPECTRUM OF A NON-SELF-ADJOINT RANDOM OPERATOR

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Abstract. The purpose of this paper is to prove that the spectrum of the non-self-adjoint one-particle Hamiltonian proposed by J. Feinberg and A. Zee (Phys. Rev. E 59 (1999), 6433–6443) has interior points. We do this by first recalling that the spectrum of this random operator is the union of the set of ℓ^∞ eigenvalues of all infinite matrices with the same structure. We then construct an infinite matrix of this structure for which every point of the open unit disk is an ℓ^∞ eigenvalue, this following from the fact that the components of the eigenvector are polynomials in the spectral parameter whose non-zero coefficients are ± 1 's, forming the pattern of an infinite discrete Sierpinski triangle.

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