

## A REALIZATION THEOREM FOR GENERALIZED NEVANLINNA FAMILIES

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*Abstract.* Boundary relations for a symmetric relation in a Pontryagin space are studied and the corresponding Weyl families are characterized. In particular, it is shown that every generalized Nevanlinna family can be realized as the Weyl family of a boundary relation in a Pontryagin space.

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