

INVERSE NODAL PROBLEMS FOR THE STURM-LIOUVILLE OPERATOR WITH EIGENPARAMETER DEPENDENT BOUNDARY CONDITIONS

CHUAN-FU YANG

Abstract. An inverse nodal problem consists in reconstructing this operator from the given zeros of their eigenfunctions. In this work, we are concerned with the inverse nodal problem of the Sturm-Liouville operator with eigenparameter dependent boundary conditions on a finite interval. We prove uniqueness theorems: a dense subset of nodal points uniquely determine the parameters of the boundary conditions and the potential function of the Sturm-Liouville equation; and provide a constructive procedure for the solution of the inverse nodal problems.

Mathematics subject classification (2010): 47A10, 47A20, 47A45, 47A67, 47B25.

Keywords and phrases: Sturm-Liouville operator, eigenparameter dependent boundary conditions, inverse nodal problem, reconstruction formula.

REFERENCES

- [1] P. A. BINDING, P. J. BROWNE, AND B. A. WATSON, *Equivalence of inverse Sturm-Liouville problems with boundary conditions rationally dependent on the eigenparameter*, J. Math. Anal. Appl. **291** (2004), 246–261.
- [2] P. J. BROWNE, B. D. SLEEMAN, *Inverse nodal problem for Sturm-Liouville equation with eigenparameter dependent boundary conditions*, Inverse Problems **12** (1996), 377–381.
- [3] S. A. BUTERIN, C. T. SHIEH, *Inverse nodal problem for differential pencils*, Applied Mathematics Letters **22** (2009), 1240–1247.
- [4] Y. T. CHEN, Y. H. CHENG, C. K. LAW AND J. TSAY, L^1 convergence of the reconstruction formula for the potential function, Proc. Amer. Math. Soc. **130** (2002), 2319–2324.
- [5] Y. H. CHENG, C. K. LAW, J. TSAY, *Remarks on a new inverse nodal problem*, J. Math. Anal. Appl. **248** (2000), 145–155.
- [6] M. V. CHUGUNOVA, *On the inverse spectral problem for the finite interval*, Funct. Analysis, Ulyanovsk **35** (1994), 113–123.
- [7] M. V. CHUGUNOVA, *On the effective methods for the solution of some inverse problems*, Funct. Analysis, Ulyanovsk **36** (1997), 66–75.
- [8] S. CURRIE, B. A. WATSON, *Inverse nodal problems for Sturm-Liouville equations on graphs*, Inverse Problems **23** (2007), 2029–2040.
- [9] G. FREILING, V. A. YURKO, *Inverse Sturm-Liouville Problems and Their Applications*, NOVA Science Publishers, New York, 2001.
- [10] C. T. FULTON, *Two-point boundary value problems with eigenvalue parameter contained in the boundary conditions*, Proc. Roy. Soc. Edinburgh **77** (A) (1977), 293–308.
- [11] O. H. HALD, J. R. MCLAUGHLIN, *Solutions of inverse nodal problems*, Inverse Problems **5** (1989), 307–347.
- [12] H. HOCHSTADT, B. LIEBERMAN, *An inverse Sturm-Liouville problem with mixed given data*, SIAM J. Appl. Math. **34** (1978), 676–680.
- [13] H. KOYUNBAKAN, *A new inverse problem for the diffusion operator*, Appl. Math. Lett. **19** (2006), 995–999.
- [14] C. K. LAW, C. L. SHEN, C. F. YANG, *The inverse nodal problem on the smoothness of the potential function*, Inverse Problems **15** (1999), 253–263; Errata **17** (2001) 361–364.

- [15] C. K. LAW, J. TSAY, *On the well-posedness of the inverse nodal problem*, *Inverse Problems* **17** (2001), 1493–1512.
- [16] C. K. LAW, C. F. YANG, *Reconstructing the potential function and its derivatives using nodal data*, *Inverse Problems* **14**, 2 (1998) 299–312.
- [17] B. JA. LEVIN, *Distribution of zeros of entire functions*, AMS Transl. Vol. 5, Providence, 1964.
- [18] B. M. LEVITAN, *Inverse Sturm-Liouville Problems*, Utrecht: VNU Science Press, 1987.
- [19] B. M. LEVITAN, I. S. SARGSIAN, *Sturm-Liouville and Dirac Operators* (Russian), Nauka, Moscow 1988; English transl., Kluwer, Dordrecht, 1991.
- [20] V. A. MARCHENKO, *Sturm-Liouville Operators and Their Applications*, Naukova Dumka, Kiev, 1977; English transl.: Birkh \ddot{a} ser, 1986.
- [21] J. R. MCLAUGHLIN, *Inverse spectral theory using nodal points as data—a uniqueness result*, *J. Differential Equations* **73** (1988), 354–362.
- [22] J. PÖSCHEL, E. TRUBOWITZ, *Inverse Spectral Theory*, Academic Press, Orlando, 1987.
- [23] C. L. SHEN, *On the nodal sets of the eigenfunctions of the string equations*, *SIAM J. Math. Anal.* **19** (1988), 1419–1424.
- [24] C. L. SHEN, C. T. SHIEH, *An inverse nodal problem for vectorial Sturm-Liouville equation*, *Inverse Problems* **16** (2000), 349–356.
- [25] C. T. SHIEH, V. A. YURKO, *Inverse nodal and inverse spectral problems for discontinuous boundary value problems*, *J. Math. Anal. Appl.* **347** (2008), 266–272.
- [26] J. WALTER, *Regular eigenvalue problems with eigenvalue parameter in the boundary conditions*, *Math. Z.* **133** (1973), 301–312.
- [27] X. F. YANG, *A solution of the inverse nodal problem*, *Inverse Problems* **13** (1997), 203–213.
- [28] V. A. YURKO, *Inverse Spectral Problems for Differential Operators and Their Applications*, Gordon and Breach, Amsterdam, 2000.
- [29] V. A. YURKO, *Integral transforms connected with discontinuous boundary value problems*, *Integral Transforms Spec. Funct.* **10**, 2 (2000), 141–164.
- [30] V. A. YURKO, *Inverse nodal problems for Sturm-Liouville operators on star-type graphs*, *J. Inv. Ill-Posed Problems* **16** (2008), 715–722.