

THE FLOW APPROACH FOR WAVES IN NETWORKS

BERND KLÖSS

Abstract. We present a “non-standard method” to treat wave equations on networks, leading to a transport process on the doubled directed graph. From the node conditions, we derive a flow governed by a certain adjacency matrix which, in particular, builds the bridge to the theory of difference operators. This approach provides the fundament for a powerful method to examine (boundary-)controllability and to prove stability results for damped and delay-damped networks of wave equations.

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