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MAPS PRESERVING PERIPHERAL SPECTRUM OF JORDAN PRODUCTS OF OPERATORS

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Abstract. Let $\mathscr{A}$ and $\mathscr{B}$ be (not necessarily unital or closed) standard operator algebras on complex Banach spaces X and Y, respectively. For a bounded linear operator A on X, the peripheral spectrum $\sigma_\pi(A)$ of A is defined by $\sigma_\pi(A) = \{z \in \sigma(A) : |z| = \max_{w\in\sigma(A)} |w|\}$, where $\sigma(A)$ denotes the spectrum of A. Assume that $\Phi : \mathscr{A} \to \mathscr{B}$ is a map and the range of Φ contains all operators with rank at most two. It is proved that the map Φ satisfies the condition that $\sigma_\pi(\Phi(A)\Phi(B)+\Phi(B)\Phi(A)) = \sigma_\pi(AB+BA)$ for all $A,B \in \mathscr{A}$ if and only if either there exists an invertible operator $T \in \mathscr{B}(X,Y)$ such that $\Phi(A) = \varepsilon TAT^{-1}$ for every $A \in \mathscr{A}$; or X and Y are reflexive and there exists an invertible operator $T \in \mathscr{B}(X^*,Y)$ such that $\Phi(A) = \varepsilon TA^*T^{-1}$ for every $A \in \mathscr{A}$, where $\varepsilon \in \{1,-1\}$. Furthermore, the same conclusion holds if $\mathscr{A}$ and $\mathscr{B}$ are replaced by standard real Jordan algebras of self-adjoint operators on complex Hilbert spaces. If X and Y are complex Hilbert space, we characterize also maps preserving the peripheral spectrum of the product AB^*+B^*A, and prove that such maps are of the form $A \mapsto \gamma UAU^*$ or $A \mapsto \gamma UA^tU^*$, where $U \in \mathscr{B}(X,Y)$ is a unitary operator and $\gamma \in \mathbb{C}$ with $|\gamma| = 1$, A^t denotes the transpose of A for an arbitrary but fixed orthonormal basis of X.

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