

STRONG COMMUTATIVITY PRESERVING MAPS ON TRIANGULAR RINGS

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Abstract. Let $\mathcal{U} = \text{Tri}(\mathcal{A}, \mathcal{M}, \mathcal{B})$ be a triangular ring. It is shown, under some mild assumption, that every surjective strong commutativity preserving map $\Phi: \mathcal{U} \rightarrow \mathcal{U}$ (i.e. $[\Phi(T), \Phi(S)] = [T, S]$ for all $T, S \in \mathcal{U}$) is of the form $\Phi(T) = ZT + f(T)$, where Z is in $\mathcal{Z}(\mathcal{U})$, the center of \mathcal{U} , $Z^2 = I$ and f is a map from \mathcal{U} into $\mathcal{Z}(\mathcal{U})$. As an application, a characterization of general surjective maps that preserve the strong commutativity on the nest algebras of Banach space operators is given.

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