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SAMUEL MULTIPLICITIES AND BROWDER SPECTRUM OF OPERATOR MATRICES

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Abstract. In this paper, we first point out that the necessity of Theorem 4 in [8] does not hold under the given condition and present a revised version with a little modification. Then we show that the definitions of some classes of semi-Fredholm operators, which use the language of algebra and first introduced by X. Fang in [8], are equivalent to that of some well-known operator classes. For example, the concept of shift-like semi-Fredholm operator on Hilbert space coincide with that of upper semi-Browder operator. For applications of Samuel multiplicities we characterize the sets of $\bigcap_{C\in B(K,H)}\sigma_{ab}(M_C), \bigcap_{C\in B(K,H)}\sigma_{sb}(M_C)$ and $\bigcap_{C\in B(K,H)}\sigma_{b}(M_C)$, respectively, where $M_C = \begin{pmatrix} A & C \\ 0 & B \end{pmatrix}$ denotes a 2-by-2 upper triangular operator matrix acting on the Hilbert space $H \oplus K$.

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REFERENCES

[1] X. H. CAO, *Browder spectra for upper triangular operator matrices*, J. Math. Anal. Appl. **342** (2008), 477–484.
[2] X. L. CHEN, S. F. ZHANG, H. J. ZHONG, *On the filling in holes problem of operator matrices*, Linear Algebra Appl. **430** (2009), 558–563.
[3] D. S. DJORDJEVIĆ, *Perturbations of spectra of operator matrices*, J. Operator Theory **48** (2002), 467–486.
[4] H. K. DU, J. PAN, *Perturbation of spectrums of* 2×2 *operator matrices*, Proc. Amer. Math. Soc. **121** (1994), 761–766.
[5] J. ESCHMEIER, *Samuel multiplicity and Fredholm theory*, Math. Ann. **339** (2007), 21–35.
[6] J. ESCHMEIER, *On the Hilbert-Samuel multiplicy of Fredholm tuples*, Indiana Univ. Math. J. **56** (2007), 1463–1477.
[7] J. ESCHMEIER, *Samuel multiplicity for several commuting operators*, J. Operator Theory **60** (2008), 399–414.
[8] X. FANG, *Samuel multiplicity and the structure of semi-Fredholm operators*, Adv. Math. **186** 2 (2004), 411–437.
[9] X. FANG, *Hilbert polynomials and Arveson's curvature invariant*, J. Funct. Anal. **198**, 2 (2003), 445–464.
[10] X. FANG, *Invariant subspaces of the Dirichlet space and commutative algebra*, J. Reine Angew. Math. **569** (2004), 189–211.
[11] X. FANG, *The Fredholm index of quotient Hilbert modules*, Math. Res. Lett. **12** (2005), 911–920.
[12] X. FANG, *The Fredholm index of a pair of commuting operators*, Geom. Funct. Anal. **16** (2006), 367–402.
[13] J. K. HAN, H. Y. LEE, W. Y. LEE, *Invertible completions of* 2×2 *upper triangular operator matrices*, Proc. Amer. Math. Soc. **128** (1999), 119–123.
[14] I. S. HWANG, W. Y. LEE, *The boundedness below of* 2×2 *upper triangular operator matrices*, Integr. Equ. Oper. Theory **39** (2001), 267–276.

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[15] W. Y. LEE, *Weyl's theorem for operator matrices*, Integr. equ. oper. theory **32** (1998), 319–331.
[16] W. Y. LEE, *Weyl spectra of operator matrices*, Proc. Amer. Math. Soc. **129** (2000), 131–138.
[17] S. F. ZHANG, H. J. ZHONG, Q. F. JIANG, *Drazin spectrum of operator matrices on the Banach space*, Linear Algebra Appl. **429** (2008), 2067–2075.
[18] S. F. ZHANG, Z. Y. WU, H. J. ZHONG, *Continuous spectrum, point spectrum and residual spectrum of operator matrices*, Linear Algebra Appl. **433** (2010), 653–661.
[19] S. F. ZHANG, H. J. ZHONG, J. D. WU, *Spectra of Upper-triangular Operator Matrices*, Acta Math. Sci. (in Chinese) **54** (2011), 41–60.

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