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# CLOSED LINEAR RELATIONS AND THEIR REGULAR POINTS

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*Abstract.* For a closed linear relation $A$ in a Hilbert space $\mathfrak{H}$ the notions of resolvent set and set of points of regular type are extended to the set of regular points. Such points are defined in terms of quasi-Fredholm relations of degree $0$. The set of regular points is open and for $\lambda \in \mathbb{C}$ in this set the spaces $\ker(A-\lambda)$ and $\operatorname{ran}(A-\lambda)$ are continuous in the gap metric. Several characterizations of regular points are presented, in terms of the gap metric between corresponding null spaces, and in terms of generalized resolvents of the linear relation $A$.

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