

APPROXIMATE INNER PRODUCTS ON HILBERT C^* -MODULES; A FIXED POINT APPROACH

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Abstract. We present a fixed point method to investigate the stability and superstability of inner products on Banach modules over a C^* -algebra. Moreover, we show that under some conditions on approximate inner product, the Banach modules over a C^* -algebra has a Hilbert C^* -module structure.

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