

A NOTE ON JORDAN DERIVABLE LINEAR MAPS

GREGOR DOLINAR, KAN HE, BOJAN KUZMA AND XIAOFEI QI

Abstract. Let H be a complex Hilbert space and let δ be a linear map which is Jordan derivable at a given idempotent $P \in B(H)$ in the sense that $\delta(A^2) = \delta(A)A + A\delta(A)$ holds for all A with $A^2 = P$. If P has infinite rank and co-rank, then we prove that the restriction of δ to $B(\text{Im}P)$ is an inner derivation and the restriction to $B(\text{Ker}P)$ is a sum of inner derivation and multiplication by a scalar. We give an example that this is not necessarily true when rank and co-rank of P are finite.

Mathematics subject classification (2010): 47B47, 47L30.

Keywords and phrases: Derivations, Jordan derivable linear maps, Hilbert spaces.

REFERENCES

- [1] E. CHRISTENSEN, *Derivations of nest algebras*, Math. Ann. **229** (1977), 155–161.
- [2] I. N. HERSTEIN, *Jordan derivation on prime rings*, Proc. Amer. Math. Soc. **8** (1957), 1104–1110.
- [3] J. HOU, X. QI, *Additive maps derivable at some points on J -subspace lattice algebras*, Linear Algebra Appl. **429** (2008), 1851–1863.
- [4] M.-Y. JIAO, J.-C. HOU, *Additive maps derivable or Jordan derivable at zero point on nest algebras*, Linear Algebra Appl. **432** (2010), 2984–2994.
- [5] W. JING, *On Jordan all-derivable points of $B(H)$* , Linear Algebra Appl. **430** (2009), 941–946.
- [6] W. JING, S. LU, *Generalized Jordan derivations on prime rings and standard operator algebras*, Taiwanese Journal of Mathematics **7** (2003), 605–613.
- [7] C. PEARCY, D. TOPPING, *Sums of small numbers of idempotents*, Michigan Math. J. **14** (1967) 453–465.
- [8] X.-F. QI, J.-C. HOU, *Characterizations of derivations of Banach space nest algebras: All-derrivable points*, Linear Algebra Appl. **432** (2010), 3183–3200.
- [9] J. ZHU, C. XIONG, *All-derivable points in continuous nest algebras*, J. Math. Anal. Appl. **340** (2008), 845–853.