

## JORDAN DERIVATIONS AND ANTIDERIVATIONS OF GENERALIZED MATRIX ALGEBRAS

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*Abstract.* Let  $\mathcal{G} = \begin{bmatrix} A & M \\ N & B \end{bmatrix}$  be a generalized matrix algebra defined by the Morita context  $(A, B, {}_A M_{B,B}, {}_B N_A, \Phi_{MN}, \Psi_{NM})$ . In this article we mainly study the question of whether there exist the so-called “proper” Jordan derivations for the generalized matrix algebra  $\mathcal{G}$ . It is shown that if one of the bilinear pairings  $\Phi_{MN}$  and  $\Psi_{NM}$  is nondegenerate, then every antiderivation of  $\mathcal{G}$  is zero. Furthermore, if the bilinear pairings  $\Phi_{MN}$  and  $\Psi_{NM}$  are both zero, then every Jordan derivation of  $\mathcal{G}$  is the sum of a derivation and an antiderivation. Several constructive examples and counterexamples are presented.

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