

## OPERATOR-VALUED FRAME GENERATORS FOR GROUP-LIKE UNITARY SYSTEMS

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*Abstract.* We investigate operator-valued frames with the structure of group-like unitary systems. We show that the commutant of a group-like unitary system can be characterized in terms of the analysis operators associated with all the operator-valued Bessel generators and give some sufficient and necessary conditions to describe when an operator-valued frame generator admits a Parseval dual. This extends work of J.Gabardo and D.Han but the proofs turn out to be more complicated. Then we characterize the Parseval operator-valued frame generators for certain unitary systems on a finite dimensional Hilbert space.

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