

\mathbb{R} -ORBIT REFLEXIVE OPERATORS

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Abstract. We completely characterize orbit reflexivity and \mathbb{R} -orbit reflexivity for matrices in $\mathcal{M}_N(\mathbb{R})$. Unlike the complex case in which every matrix is orbit reflexive and \mathbb{C} -orbit reflexivity is characterized solely in terms of the Jordan form, the orbit reflexivity and \mathbb{R} -orbit reflexivity of a matrix in $\mathcal{M}_N(\mathbb{R})$ is described in terms of the linear dependence over \mathbb{Q} of certain elements of \mathbb{R}/\mathbb{Q} . We also show that every $n \times n$ matrix over an uncountable field \mathbb{F} is algebraically \mathbb{F} -orbit reflexive.

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