

SINGULAR WEYL–TITCHMARSH–KODAIRA THEORY FOR JACOBI OPERATORS

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Abstract. We develop singular Weyl–Titchmarsh–Kodaira theory for Jacobi operators. In particular, we establish existence of a spectral transformation as well as local Borg–Marchenko and Hochstadt–Liebermann type uniqueness results.

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