

BIVARIATE MATRIX FUNCTIONS

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Abstract. A definition of bivariate matrix functions is introduced and some theoretical as well as algorithmic aspects are analyzed. It is shown that our framework naturally extends the usual notion of (univariate) matrix functions and allows to unify existing results on linear matrix equations and derivatives of matrix functions.

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