

## LOWER BOUNDS ON THE NORMS OF EXTENSION OPERATORS FOR LIPSCHITZ DOMAINS

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*Abstract.* Let  $\Omega \subset \mathbb{R}^d$  be a bounded or an unbounded Lipschitz domain. In this note we address the problem of continuation of functions from the Sobolev space  $H^1(\Omega)$  up to functions in the Sobolev space  $H^1(\mathbb{R}^d)$  via a linear operator. The minimal possible norm of such an operator is estimated from below in terms of spectral properties of self-adjoint Robin Laplacians on domains  $\Omega$  and  $\mathbb{R}^d \setminus \overline{\Omega}$ . Another estimate of this norm is also given, where spectral properties of Schrödinger operators with the  $\delta$ -interaction supported on the hypersurface  $\partial\Omega$  are involved. General results are illustrated with examples.

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