

COMMUTATION RELATIONS FOR TRUNCATED TOEPLITZ OPERATORS

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Abstract. For truncated Toeplitz operators, which are compressions of multiplication operators to model subspaces of the Hardy space H^2 , we obtain criteria for commutation relations. The results show an analogy to the case of Toeplitz matrices, and they extend the theory of Sedlock algebras.

Mathematics subject classification (2010): 47B32, 47B35, 47B37.

Keywords and phrases: Truncated Toeplitz operators, normal operators, commutation properties.

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