Operators
and
Matrices
Volume 8, Number 3 (2014), 889–899
doi:10.7153/oam-08-50

STRONG CONTINUITY OF THE LIDSTONE EIGENVALUES OF THE BEAM EQUATION IN POTENTIALS

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Abstract. In this paper we study the dependence of the Lidstone eigenvalues $\lambda_m(q)$, $m \in \mathbb{N}$, of the fourth-order beam equation on potentials $q \in L^p[0,1]$, $1 \leqslant p \leqslant \infty$. The first result is that $\lambda_m(q)$ have a strongly continuous dependence on potentials, i.e., as nonlinear functionals, $\lambda_m(q)$ are continuous in $q \in L^p[0,1]$ when the weak topology is considered. The second result is that $\lambda_m(q)$ are continuously Fréchet differentiable in potentials $q \in L^p[0,1]$ when the L^p norm is considered. These results will be used in studying the optimal estimations for these eigenvalues in later works.

Mathematics subject classification (2010): 34L40, 34A30, 58C07.
Keywords and phrases: Beam equation, eigenvalue, strong continuity, weak topology, Fréchet differentiability.

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